Aristotelian Ontology and Modal Syllogistic Reconstructed

Yutaka TANAKA*

The following system of ontology recommended itself to us in the first instance by its ability to reformulate certain arguments of medieval scholastics, of which the one best known to historians of metaphysics is the theory of transcendentals. But the system in question seems not only useful to historical exegesis; it can also, if we are not mistaken, suggest an answer to the current problems of modal logic. Modern logicians unanimously admit that there is a great affinity between modal logic and Aristotelian "essentialism."[1] The concept of essence, however, is thought to be so much opaque that the very plausibility of modal logic tends to become a controversial issue. Our system contains a reconstruction of syllogistic theory which enables us to discriminate the ambiguities of modal premises. The modal syllogism is not only an extension of simple syllogistic formula, but also it involves an explicit ontological commitment when it uses the concept of essential equality between two terms. On the other hand, the simple syllogism is readily susceptible to ontologically neutral axiomatization. This is the reason why many textbooks of formal logic exclude modality from syllogistic theory, saying, "De modalibus non gustabit asinus."[2]

According to Aristotle (Met. 1005b5-10) it belongs to the philosopher, *i. e.* to him who is studying the nature of all substance, to inquire into the principles of syllogism. But he who knows best about each genus must be able to state the most certain principles of his subject, so that he whose subject is existing things qua existing must be able to state the most certain principles of all things. Now there is a science which investigates being as being (to on hêi on) and the attributes which belong to this in virtue of its own nature. (Met. 1003a18) Aristotle was the founder of this science *i. e.* the general ontology.

Among the fourfold classification of different senses of "being" which Aristotle mentioned (Met. 1026a33-b2), the third as well as the fourth plays an important role in metaphysics *i. e.* "being of the categories" and "being in potentiality and actuality."[3] According to his paradigm we start off by choosing two basic formulae

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[1] Quine says, "The way to do quantified modal logic, if at all, is to accept Aristotelian essentialism." To defend Aristotelian essentialism, however, is not his plan. Such a philosophy is as unreasonable by his lights as it is by Carnap's or Lewis's.
Reference and Modality, ed. by Leonard Linsky, p. 31.

[2] Storrs MaCall, *Aristotle's Modal Syllogisms*, p. 1. On the history of modal logic, see the following literatures; W. Kneale, *The Development of Logic*, pp. 81–96, 117–128, 548–568, 628–652. J. M. Bochenski, *Formale Logik*, pp. 94–100, 116–118, 131–133, 260–267, 467–471.

[3] Franz Brentano, *On the several Senses of Being in Aristotle*, tr. by Rolf George, pp. 3, 28, 52 ff.

from chap. 2 of Categoriae, and proceed to define some fundamental concepts of general ontology so that we may discuss the ontological aspects of modal syllogistic and establish the most universal laws of what there is.

(A) Presentation of Aristotelian Ontology as an Axiomatized System

(A-1) Two basic formulae of predication and existence

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(a) *Predicative Formula*: universal *vs.* particular:
Ka (A, B): A is said of the subject B.
(to A kath' hypokeimenou toû B legetai)
(b) *Existential Formula*: abstract *vs.* concrete:
En (A, B): A is in the subject B.
(to A en hypokeimenôi tôi B estin)

Ka (A, B) means either that A is predicated of the individual B, or that A is predicated of every B. (predicative presupposition)
En (A, B) means either that A exists in the individual B, or that A exists in some B. (existential presupposition)[4]

Examples of Ka (A, B) and En (A, B)

(i) Aristotle (Categoriae 1a20-b9)
Ka (A, B):
Ka (to zôion, ho anthrôpos) *i. e.* Ka (animal, man)
Ka (ho anthrôpos, ho tis anthrôpos) *i. e.* Ka (man, the individual man)
Ka (hê epistêmê, hê grammatikê) *i. e.* Ka (knowledge, the knowledge of grammar)
En (A, B):
En (to ti leukon, to sôma) *i. e.* En (the individual whiteness, body)
En (hê tis grammatikê, hê psychê) *i. e.* En (the individual knowledge of grammar, soul)
En (hê epistêmê, hê pschê) *i. e.* En (knowledge, soul)

(ii) Ka (A, B) brings a partial order among entities of which the most famous example is the so-called arbor Porphyrii (Isagoge 2a13-42):
Ka (substance, body) Ka (body, living body) Ka (living body, animal)
Ka (animal, rational animal) Ka (rational animal, man) Ka (man, Socrates)

(iii) Ka (A, B), as (i) and (ii) show, contains the predication per se *i. e.* the relation of genus to species or that of species to the individual.
We add further the analytical predication as an example of Ka (A, B), so that Ka (A, B) may be an existentially free formula:
Ka (round, round square) Ka (golden, golden mountain)

[4] We had better use the word "presupposition" in order to express what Aristotle means by "hypokeimenon" (Cat. 1a20). *cf.* Tadashi Inoue, *Tetsugaku no Genba* (Groundwork of Philosophy), pp. 146–164.

(A-2) Axiomatization of the fundamental principles of predication and existence

(axiom-1) *The Law of Transitive Predication*
Ka (A, B)∧Ka (B, C)→Ka (A, C)

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(axiom-3) *The Law of Existential Presupposition*
En (A, B)∧Ka (C, B)→En (A, C)

Aristotle mentioned the first axiom in Categoriae 1b10-16: hotan heteron kath' heterou katêgorêtai hôs kath' hypokeimenou, hosa kata toû katêgoroûmenou legetai, panta kai kata toû hypokeimenou rêthêsetai. (When one thing is predicated of another, all that which is predicated of the predicate will be predicated also of the subject.)

Immanuel Kant looked on this axiom as the highest principle of syllogism in his dissertation "Die falsche Spitzfindigkeit der vier syllogistischen Figuren" (B53-61):
Die erste und allgemeine Regel aller bejahenden Vernunftschlusse sei: Ein Merkmal vom Merkmal ist ein Merkmal der Sache selbst (nota notae est etiam nota rei ipsius); von allen verneinenden: Was dem Merkmal eines Dinges wiederspricht, widerspricht dem dinge selbst (repugnans notae repugnat rei ipsi).

In our system the negative principle (repugnans notae repugnat rei ipsi) is represented by the formula Ka (Ā, B)∧Ka (B, C)—Ka (Ā, C), which will be a special case of the first axiom.

The second axiom states, roughly speaking, that an abstract universal attribute exists in the concrete particular thing by the medium of an abstract particular attribute. En (A, B) provides every entity with concrete basis of existence.[5]

On the other hand the formula that Ka (A, B)∧En (B, C)—Ka (A, C) doesnot hold good. Take the following example as a false syllogism:

The red is a colour.	Ka (colour, redness)
This apple is red.	En (redness, this apple)
Therefore,	Therefore,
This apple is a colour.	Ka (colour, this apple)

We must take notice of a linguistic peculiality of Greek: "the red" can mean both the red thing and redness itself. If En (A, B) holds good, then A must designate an abstract attribute, and not the entity which is A. Aristotle seemed to be aware of this ambiguity, when he said, "When a thing is in the subject, though the name

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In our system En (A, B) means more than inherence of attributes in an individual substance; it also means an existential presupposition, which is the reason why (axiom-3) is added. There is a controversial issue concerning the status of individual attribute in Categoriae. *cf.* G. E. L. Owen, "Inherence", *Phronesis*, 10, 1965, pp. 97–105.

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may quite well be applied to that in which it exists, the explanatory formula cannot be applied." (Cat. 3a15)

While the more concrete has a kind of priority over the more abstract in the context of existence, the latter has another kind of priority over the former in the context of predication. For example, a man is called brave (andreîos) because the bravery (hê andreia) exists in him. (an example of paronym. *cf.* Cat. 1a15)

The third axiom states that if A is in the subject B, and C is said of the subject B, then A is in the subject C, *i. e.* were it not for C, A would not exist at all.

For example,

En (knowledge, the individual soul)
Ka (soul, the individual soul)

Therefore, En (knowledge, soul)

It must be remembered that there is no type-difference between concept and object in our system: both ordered pairs "animal-man" and "man- the individual man" are examples of Ka (A, B). In our system A, B, C, . . designate entities in the widest sense: whatever can be said of is represented by them.

(A-3) Defintion of equality between two terms

Our system presupposes only two formulae, Ka (A, B) and En (A, B), in terms of which we can define equality between two terms. Equality must be distinguished from equivalence: the former belongs to ontology while the latter only to formal logic. The following definition of equality mentions "all" entities which are presupposed in our system. while the definition of equivalence only mentions "any" entity without ontological commitment.

(Df-1) *The Definition of Equality*
$A=B\rightleftarrows((\forall X)\,(\mathrm{Ka}\,(A, X)\rightleftarrows\mathrm{Ka}\,(B, X))\wedge(\forall X)\,(\mathrm{Ka}\,(X, A)\rightleftarrows\mathrm{Ka}\,(X, B))\wedge$
$(\forall X)\,(\mathrm{En}\,(A, X)\rightleftarrows\mathrm{En}\,(B, X))\wedge(\forall X)\,(\mathrm{En}\,(X, A)\rightleftarrows\mathrm{En}\,(X, B)))$[6]

(Df-2)
$\mathrm{ka}\,(A, B)\rightleftarrows(\mathrm{Ka}\,(A, B)\wedge(A\neq B))$
$\mathrm{en}\,(A, B)\rightleftarrows(\mathrm{En}\,(A, B)\wedge(A\neq B))$

(Df-3) *The Definition of Equivalence*
$A\equiv B\rightleftarrows(\mathrm{Ka}\,(A, B)\wedge\mathrm{Ka}\,(B, A))$

We say that A and B are convertible terms when $A\equiv B$ in the above sense.

(A-4) Introduction of term-negation and term-connectives to our system

Term-negation: $\bar{A}$ (non-A)
Formula-negation: $\sim p$ (it is not the case that p)
Term-connectives: A & B (both A and B), A or B (either A or B)
Formula-connectives: $p\wedge q$ (it is the case that p, and it is the case that q)

[6] This resembles what Leibniz called the identity of indiscernibles. *cf.* Bertrand Russell, *The Philosophy of Leibniz*, pp. 54–69.

$p\vee q$ (it is the case that p, or it is the case that q)

We follow the ordinary usage of formula-negation and formula-connectives in propositional logic. As for term-negation and term-connectives, we must specify the postulates which regulates term operations.

(post-1) $A=B\rightarrow\bar{A}=B$ (post-2) $\bar{\bar{A}}=A$ (post-3) $A=B\rightarrow A\&C=B\&C$
(post-4) $A\&B=B\&A$ (post-5) $A\&A=A$ (post-6) $(A\&B)\&C=A\&(B\&C)$
(Df-4) $A\mathrm{or}B=\overline{A\&B}$

The above postulates involve ontological commitment because of equality; they are extra-logical principles. We add the following axioms which are important to the concept of analyticity.

(axiom-4) *The Law of Analytical Predication*[7]
Ka (A, A&B)
(axiom-5) *The Law of Conjunction*
$\mathrm{Ka}\,(A, C)\wedge\mathrm{Ka}\,(B, C)\rightarrow\mathrm{Ka}\,(A\&B, C)$

We can easily deduce from (axiom-1), (axiom-4), and propositional logic, that $\mathrm{Ka}\,(A\&B, C)\rightarrow\mathrm{Ka}\,(A, C)\wedge\mathrm{Ka}\,(B, C)$.

On the contrary, "$\mathrm{Ka}\,(A\mathrm{or}B, C)\rightarrow\mathrm{Ka}\,(A, C)\vee\mathrm{Ka}\,(B, C)$" doesn't hold good. For counter-example, "Ka (odd or even, number)" is true, while "Ka (odd, number) $\vee$ Ka (even, number)" is false.

It directly follows from (axiom-4) and (post-5) that Ka (A, A).
(*i. e.* the law of identical predication)

(A-5) Axiomatization of simple syllogistic without existential import

We can difine syllogistic premises in terms of Ka (A, B) in the following way:
(Df-5)
All B are A (without existential import): $AaB\rightleftarrows\mathrm{Ka}\,(A, B)$
No B are A: $AeB\rightleftarrows\mathrm{Ka}\,(\bar{A}, B)$
Some B are A: $AiB\rightleftarrows\sim\mathrm{Ka}\,(\bar{A}, B)$
Some B are not A: $AoB\rightleftarrows\sim\mathrm{Ka}\,(A, B)$
(Df-6)
All B are A (with existential import): $Aa'B\rightleftarrows AaB\wedge BiB$

We can prove all valid syllogisms from Barbara and E-conversion, if we admit the rules of inference of propositional logic.

The following system of simple syllogism is a subsystem of general ontology, and the distinction of validity with or without existential presupposition is drawn clearly.[8]

(axiom-1) *The Law of Transitive Predication*
$\mathrm{Ka}\,(A, B)\wedge\mathrm{Ka}\,(B, C)\rightarrow\mathrm{Ka}\,(A, C)$ *i. e.* $AaB\wedge BaC\rightarrow AaC$ (Barbara)
(axiom-6) *The Law of Predicative Exclusion*
$\mathrm{Ka}\,(\bar{A}, B)\rightarrow\mathrm{Ka}\,(\bar{B}, A)$ *i. e.* $AeB\rightarrow BeA$ (E-conversion)

[7] Immanuel Kant, *Kritik der reinen Vernunft*, Einleitung IV, B 11. His distinction between analytical and synthetic judgement is only made clear through term-connectives.

The Rules of Inference: (p, q, r, s,—designate formula variables)

(R1) $\dfrac{p \land q \to r \qquad r \to s}{p \land q \to s}$ (R2) $\dfrac{p \land q \to r \qquad s \to p}{s \land q \to r}$

(R3) $\dfrac{p \land q \to r}{q \land p \to r}$ (R4) $\dfrac{p \land q \to r}{q \land \sim r \to \sim p}$ (R5) $\dfrac{p \to q}{\sim q \to \sim p}$

(R6) The Rule of Substitution for Formula Variables

(R7) The Rule of Substitution for Term Variables

The Proof of Celarent:

(1) $\mathrm{Ka}(\bar{A}, B) \land \mathrm{Ka}(B, C) \to \mathrm{Ka}(\bar{A}, C)$ ... (axiom-1) (R7)
(2) $AeB \land BaC \to AeC$ ... (1) (Df-5)

The Proof of I-conversion:

(1) $\mathrm{Ka}(\bar{B}, A) \to \mathrm{Ka}(\bar{A}, B)$ ... (axiom-6) (R7)
(2) $\sim \mathrm{Ka}(\bar{A}, B) \to \sim \mathrm{Ka}(\bar{B}, A)$ ... (R5) (R6) (1)
(3) $AiB \to BiA$ ... (2) (Df-5)

The Proof of Darii:

(1) $\mathrm{Ka}(\bar{C}, A) \land \mathrm{Ka}(A, B) \to \mathrm{Ka}(\bar{C}, B)$ ... (axiom-1) (R7)
(2) $\mathrm{Ka}(\bar{C}, B) \to \mathrm{Ka}(\bar{B}, C)$ ... (axiom-6) (R7)
(3) $\mathrm{Ka}(\bar{C}, A) \land \mathrm{Ka}(A, B) \to \mathrm{Ka}(\bar{B}, C)$ ... (1) (2) (R1) (R6)
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(7) $\mathrm{Ka}(A, B) \land \sim \mathrm{Ka}(\bar{B}, C) \to \sim \mathrm{Ka}(\bar{A}, C)$ ... (4) (6) (R1) (R6)
(8) $AaB \land BiC \to AiC$ ... (7) (Df-5)

The Proof of Ferio:

(1) $\mathrm{Ka}(\bar{B}, A) \land \mathrm{Ka}(A, C) \to \mathrm{Ka}(\bar{B}, C)$ ... (axiom-1) (R7)
(2) $\mathrm{Ka}(\bar{A}, B) \to \mathrm{Ka}(\bar{B}, A)$ ... (axiom-6)
(3) $\mathrm{Ka}(\bar{A}, B) \land \mathrm{Ka}(A, C) \to \mathrm{Ka}(\bar{B}, C)$ ... (1) (2) (R2) (R6)
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(5) $\mathrm{Ka}(\bar{A}, B) \land \sim \mathrm{Ka}(\bar{B}, C) \to \sim \mathrm{Ka}(A, C)$ ... (4) (R4) (R6)

[8] Former examples of axiomatization of the simple syllogistic are as follows; Jan Lukasiewicz, *Aristotle's Syllogistic*, p. 88:
(axiom-1) $Aa'A$ (axiom-2) AiA (axiom-3) $Aa'B \land Ba'C \to Aa'C$ (Barbara) (axiom-4) $Aa'B \land CiB \to AiC$ (Datisi)
G. E. Hughs and D. G. Londey, *The Elements of Formal Logic*, pp. 353–361. (axiom-1) $Aa'B \land CiB \to AiC$ (Datisi) (axiom-2) $AeB \to \bar{A}a'B$ (axiom-3) $\bar{A}eA$ (axiom-4) AiA
It is somewhat clumsy for them to use "Aa'B" in the axioms, because it is not a single formula; if we replace Aa'B by AaB, we can reduce the number of the axioms to only two, as shown in our system. Moreover, Lukasiewiz's system and Hughs and Londey's don't teach us that Darapti and Felapton are invalid without existential import, while the other syllogisms which Aristotle mentioned in Prior Analytics prove to be valid without any existential condition.
In our system the valid syllogisms with respect to AaB prove to be valid when we replace AaB by the stronger premise, Aa'B, because of (R2).

(6) $AeB \land BiC \to AoC$ ... (5) (Df-5)

The Proof of Subaltern:

(1) $AaB \land BiB \to AiB$ ... (Darii)
(2) $Aa'B \to AiB$ ... (1) (Df-6)

The Proof of accidental Conversion:

(1) $Aa'B \to AiB$ ... (subaltern)
(2) $AiB \to BiA$ ... (I-conversion)
(3) $Aa'B \to BiA$ ... (1) (2) (R1) (R6)

(A-6) The definition of primary substance

According to the fourth axiom (the law of analytical predication), we get "Ka (round, round square)", "Ka (mountain, gloden mountain)", *etc.*: both "The round square is round" and "The golden mountain is a mountain" are admissible formulae when "is" in question is taken as predicative.

But we also want to say that the round square doesn't exist, and that the golden mountain doesn't exist.

The concept of primary substance is crucial when we specify existential conditions of various terms.

The principles with existential import are called hypotheses, and distinguished hereafter from axioms.

Four hypotheses are explicitly stated by Aristotle. (Cat. 1a20-b10)

(hyp-1) $(\exists A)(\forall X)(\sim \mathrm{ka}(A, X) \land (\forall X)(\sim \mathrm{en}(A, X)))$
(hyp-2) $(\exists A)(\exists X)(\mathrm{ka}(A, X) \land (\forall X)(\sim \mathrm{en}(A, X)))$
(hyp-3) $(\exists A)(\forall X)(\sim \mathrm{ka}(A, X) \land (\exists X)(\mathrm{en}(A, X)))$
(hyp-4) $(\exists A)(\exists X)(\mathrm{ka}(A, K) \land (\exists X)(\mathrm{en}(A, X)))$
(Df-7) $\mathrm{Ps}(A) \rightleftarrows (\forall X)(\sim \mathrm{ka}(A, X)) \land (\forall X)(\sim \mathrm{en}(A, X))$
Ps (A): A is a primary substance.
(Df-8) $\mathrm{S}(A) \rightleftarrows (\exists X)(\mathrm{ka}(A, X)) \land (\forall X)(\sim \mathrm{en}(A, X))$

We cannot define a secondary substance as S (A). If A is a secondary substance, then S (A); but not vice versa. S (two-footed) is a counter-example. It will be shown that the criterion of secondary substance needs more conditions which involve modality.

(Df-9) $\mathrm{Pa}(A) \rightleftarrows (\forall X)(\sim \mathrm{ka}(A, X)) \land (\exists X)(\mathrm{en}(A, X))$
Pa (A): A is a primary attribute.
(Df-10) $\mathrm{Sa}(A) \rightleftarrows (\exists X)(\mathrm{ka}(A, X)) \land (\exists X)(\mathrm{en}(A, X))$
Sa (A): A is a secondary attribute.

(Df-9) and (Df-10) are added because of symmetry, though they are not Aristotle's own terminology.

From (Df-1) we can identify a primary attribute only with reference to primary substances. For example, if En (the individual redness, the individual apple), the individual instance of redness may be different from other instances of the same colour just because it inheres in the different primary substance.

(A-7) Existential conditions with reference to primary substance

(Df-11) $\text{Exist}(A) \rightleftarrows AiA$
Exist (A): A exists.
(post-7) $\text{Ps}(A) \rightarrow \text{En}(A, A)$
i. e. if A is a primary substance, then it is in itself.
(hyp-5) $\text{Exist}(A) \rightleftarrows (\exists X)\,((\text{En}(A, X) \vee \text{Ka}(A, X)) \wedge \text{Ps}(X))$

It follows from (post-7) and (hyp-5) that the primary substances always satisfy the existential conditions, and other types of entities exist when and only when they are in some primary substances, or they are said of some primary substances.

The opposition of subalterns, accidental conversion, and such syllogisms as Darapti and Felapton are valid only when they are supplemented with existential conditions. We can use Aa'B instead of AaB in order to obtain valid inference. An example of Darapti which shows that we need an existential condition is as follows:

All golden mountains are golden.	Ka (golden, golden mountain)
All golden mountains are mountains.	Ka (mountain, golden mountain)
Therefore, some mountains are golden.	Exist (golden mountain)

Two premises of the above example are analytically true, while the conclusion is synthetically false. Darapti is an incomplete syllogism in Aristotelian ontology.[9]

(A-8) The limit concept of general ontology: transcendence and emptiness

There are two concepts which provide our system with the limit of discourse: transcendence and exptiness.

The theory of transcendentals was the main theme of medieval metaphysics. The schoolmen understood by "transcendentals" those abstract yet very real concepts which escape classification in the Aristotelian categories by reason of their greater extension and universality of application.[10] Aristotle recognized the transcendency of "being" when he said (Met. 1060b4), "Being is predicated of all things." The

[9] There seem to be three kinds of criterion in view of which a given syllogism may be judged to be complete;
(1) psychological. .*i. e.* whether it is intuitively clear or not.
(2) logical. .*i. e.* whether it is used as an axiom or not.
(3) ontological. .*i. e.* whether it is valid without any existential condition or not.
Though the first and the second have frequently been pointed out, the third, so far as I know, has never been mentioned by commentators of Aristotelian logic. The ontological criterion is discussed in my paper, "Aristotle's view of geometry" (*The Journal of Philosophy of Science Society, Japan*, Vol. 15, pp. 91–106).
There I show some evidence that Aristotle knew the difference of criterion with or without existential presupposition when he distinguished hypotheses from definitions among the principles of deductive sciences.
The problem of existential import was not ignored by medieval logicians; *cf.* Paul Thom, *The Syllogism*, pp. 114.

[10] Allan B. Wolter, *The Transcendentals and Their Function in the Metaphysics of Duns Scotus*, pp. 4–13.

transcendency of "one" was the main issue of dialectic of Plato's Parmenides, which became the Bible of Neoplatonism.

Duns Scotus listed four kinds of transcendentals as follows: (Opus Oxoniense 597b-598a)

(a) ens (b) passiones entis simpliciter convertibiles: *e. g.* unum, verum, bonum, *etc.* (c) passiones disjunctae: *e. g.* necesse esse vel possibile, actus vel potentia, *etc.* (d) perfectiones simpliciter.

The concept of emptiness was absent from Aristotelian exegeses, but it will be treated on a par with that of transcendence in our system.

(Df-12) $t(A) \rightleftarrows (\forall X)\,\text{Ka}(A, X)$
t (A): A is transcendental.

Transcendence is defined in such a way that if A is transcendental, then A is said of everything, and *vice versa*.

It follows from this definition that if A and B are transcendental, then they are convertible with each other; *i. e.*

$$t(A) \wedge t(B) \rightarrow A \equiv B$$

The above is a reformulation of scholastic thesis such as "ens et unum convertuntur."

(Df-13) $\phi(A) \rightleftarrows (\forall X)\,\text{Ka}(X, A)$
ϕ (A): A is empty.

Emptiness is defined in such a way that if A is empty, then everything is said of A, and *vice versa.*

It follows from (axiom-6) (Df-12) (Df-13) that $\bar{A}$ is empty if and only if A is transcendental, and that $\bar{A}$ is transcendental if and only if A is empty; *i. e.*

$\vdash\ \phi(\bar{A}) \rightarrow t(A) \qquad \vdash\ t(\bar{A}) \rightarrow \phi(A)$

The relation of Ka(A, B) to ϕ (A) is specified by the following hypothesis;

(hyp-6) $\text{Ka}(A, B) \rightleftarrows \phi(\bar{A}\&B)$
i. e. A is said of the subject B if and only if $\bar{A}$&B is empty.

It follows from (post-5) (axiom-4) (hyp-6) that $\bar{A}$&A is empty; from (Df-4) and the above results that Aor$\bar{A}$ is transcendental; *i. e.*

$\vdash\ \phi(\bar{A}\&A)$: *The Simple Law of Contradiction*
$\vdash\ t(A\text{or}\bar{A})$: *The Simple Law of the Excluded Middle*

We can prove that A is empty if and only if A doesn't exist in the sense of (Df-11):

$\vdash\ \phi(A) \rightleftarrows \sim\text{Exist}(A)$

(A-9) The ambiguity of transcendentals

Duns Scotus looked upon disjunctive attributes as a kind of transcendentals. It is true to say that everything is Aor$\bar{A}$, but we must take notice that this kind of statement cannot be made in a univocal sense. Aor$\bar{A}$ is equivocal, as will be seen later, when it is applied to entities of different categories.

Generally speaking, A is always equivocal when A is univocal; we cannot say in the same sense that Socrates is not white, and that Justice is not white. If a term is univocally said, then it must have generic or specific unity. Aristotle's analysis of univocals and equivocals (Cat. 1a) is based on the idea of unity which is shared by things. The categories are the highest genus that can be said univocally so that they provide the most basic framework of classification of entities.

Our next task is as follows: what is the criterion of generic unity? Genus is certainly a kind of aggregate, but the aggregate with mere enumerative unity is not always a genus. So we must define what genus is in such a way that transcendental terms such as "being" and "one" don't constitute a genus.

We proceed to define what is called trans-categorial ambiguity in our system. The laws of logic such as that of the excluded middle owe their universality to trans-categorial ambiguities.

(A-10) The definition of genus and category

Aristotle says (Topica 144a36-b11) that when the genus G is divided into the species E by the differentia D, G is not predicable of D. This is the reason why he argues (Met. 998b25) that neither "being" nor "one" constitutes a genus. So we specify the conditionship of genus in the following way:

(Df-14) $\mathrm{ge}\,(G, E) \rightleftarrows (\exists D)\,(E = G \& D \wedge \mathrm{ka}\,(G, E) \wedge \sim \mathrm{Ka}\,(G, D))$
ge (G, E): G is E's genus.

(Df-15) $\mathrm{g}\,(G) \rightleftarrows (\exists E)\,\mathrm{ge}\,(G, E)$
g (G): G is a genus.

(hyp-7) $(\exists X)\,\mathrm{g}\,(X)$

It follows from (Df-12) (Df-15) that if A is transcendental, then A is not a genus; *i.e.* $\vdash \mathrm{t}\,(A) \rightarrow \sim \mathrm{g}\,(A)$

(Df-16) $\mathrm{Ct}\,(A) \rightleftarrows \mathrm{g}\,(A) \wedge ((\forall X)\,(\mathrm{g}\,(X) \rightarrow \sim \mathrm{ka}\,(X, A)))$
Ct (A): A is a category.

(hyp-8) $(\exists X)\,\mathrm{Ct}\,(X)$

Though a single category must not be subsumed under any other genus, it need not have allinclusive universality. The categories are the highest genus which tell us what kinds of entity at bottom the various nameable things are. It is the totality of categories and not a single instance of them that has universal application. It is no wonder that Aristotle's table of categories contains "posture" and "possession" in spite of particularity.[11]

(A-11) The definition of trans-categorial ambiguity

(Df-17) $\mathrm{u}\,(B, C) \rightleftarrows (\exists X)\,(\mathrm{Ct}\,(X) \wedge \mathrm{Ka}\,(X, B) \wedge \mathrm{Ka}\,(X, C))$
u (B, C): B and C belong to the same category.

(Df-18) $\mathrm{am}\,(A, B, C) \rightleftarrows (\sim \mathrm{u}\,(B, C)) \wedge \mathrm{Ka}\,(A, B) \wedge \mathrm{Ka}\,(A, C)$
am (A, B, C): A is ambiguously said of B and of C because of category difference.

[11] Bonitz, *Über die Kategorien des Aristoteles*, chap. 2.

(Df-19) $\mathrm{tam}\,(A) \rightleftarrows (\exists B, C)\,\mathrm{am}\,(A, B, C)$
tam (A): A has a trans-categorial ambiguity.

(A-12) The definition of equivocity

(Df-20) $\mathrm{ug}\,(B, C) \rightleftarrows (\exists X)\,(\mathrm{g}\,(X) \wedge \mathrm{Ka}\,(X, B) \wedge \mathrm{Ka}\,(X, C))$
ug (B, C): B and C belong to the same genus.

(Df-21) $\mathrm{eq}\,(A, B, C) \rightleftarrows (\sim \mathrm{ug}\,(B, C)) \wedge \mathrm{Ka}\,(A, B) \wedge \mathrm{Ka}\,(A, C)$
eq (A, B, C): A is equivocally said of B and of C.

(A-13) The place of modal logic in general ontology

Aristotle discussed modality on the two levels: of which one is the theory of modal syllogism, and the other is the form/matter antithesis in his physical treatises. Though the modal syllogistic in Prior Analytics has many defects, the concept of modality plays an important role in general ontology. It is only through modal analysis that the distinction between substance and attribute is made clear; substance has a kind of descriptive priority over attributes in modal contexts.

Suppose that there is a man. We can talk of him that he may be white, he may be courageous, he may be wise, or he may go to the Lyceum. But can we talk of him—an individual man—that he can be anything other than man? Aristotle would reply that we can not; man is a secondary substance, and if anyone is a man, he is not able to be otherwise than a man, *i. e.* he must be a man. The distinction between actuality and necessity would seem to disappear in the case of secondary substances as predicates. Of course those who believe in the immortality of soul and in metempsychosis would reply that we can talk of a man that he may become an ass or other such beast in the next life. But in that case it is a soul and not a man that is a secondary substance. It may be true that they can talk of an individual soul that he is able to become something other than man, but it is certain that even they cannot talk of him that he is able to be anything other than soul.

The above thought-experiment shows that the concept of substance is indispensable for everyone no matter what opinion they may hold about life.

So our next task is as follows;

(a) To reconstruct modal logic in order that we may talk of modality de re in general ontology.
(b) To define what a secondary substance is in terms of modality de re.
(c) Axiomatization of modal syllogistic.

(A-14) Reconstruction of modal logic

We have seen that the simple syllogistic can be founded on the basis of Barbara and E-conversion. In the case of modal syllogistic the situation is far more complex. Many logicians have pointed out that Barbara with contingent modality doesn't hold good though Aristotle seems to admit it as a complete syllogism.[12] The very validity of E-conversion or of I-conversion is dubious in modal contexts.

Consider the following examples;

(a) All Russians may be Christians. (major premise)
All unbelievers may be Russians. (minor premise)

All unbelievers may be Christians. (conclusion)

(b) Some politicians must be men. (premise)

Some men must be politicians. (conclusion)

We cannot judge whether the above reasoning is valid or not before we analyse the ambiguities of modal premises.

There are at least four types of modal premises in our system. According to the type-0 of modality, the example (a) is invalid. The type-0, as will be seen later, represents a modal premise as analytical proposition; the truth or falsity of the premise can be determined through conceptual analysis. If we define a Christian as man who believes in God and Christ, then it is impossible for any Christian to be an unbeliever. On the other hand, the two premises of (a) are conceptually true; for it is compatible for anyone to be both a Christian and a Russian no less than for anyone to be both an unbeliever and a Russian. As for the example (b), the conclusion will become false if we take the type-2 interpretation, according to which it is a matter of contingence for any man to become a politician or to become, so to say, a logician. But it is a matter of necessity for any individual man to be a man.

We cannot accept Barbara and the rules of conversion uncritically because of modal ambiguity. Therefore we start off by introducing the modality as the term-operator so that we may analyse the deep structure of modal premises. The following system of modal logic is different from the ordinary ones which use the propositional operator in order to signify modality.

(A-15) Axiomatization of modal system of syllogism

We introduce the modal term-operator to our system, and specify the new postulates which regulate modal term operation.

$\Box A$ as "necessary A" or "that which must be A"
$\Diamond A$ as "possible A" or "that which may be A"

(post-8) $A=B \rightarrow \Box A=\Box B$

(post-9) $\Box\Box A=\Box A$

(post-10) $\Box(A\&B)=\Box A\&\Box B$

(post-11) $\Diamond A=\overline{\Box\overline{A}}$

The modal syllogistic requires us to use the principle of substitutivity of equals, as is shown by (post-8), while the simple syllogistic can go without any use of equality between two terms. The concept of equality is, so to say, stronger than modal operation, and it is the main characteristic of our system that various types of equivalence are defined in such a way that the substitutivity of equivalents doesn't always hold

[12] T. Sugihara, "Necessity and Possibility in Aristotelian Syllogistic", *Memoirs of the Liberal Arts College*, Fukui University, 6 & 7.

good. It must be noticed that the use of equality always involves an ontological commitment because of (Df-1) in our system.

(axiom-7) $Ka\ (\Diamond A, A)$

(axiom-8) $Ka\ ((\Diamond A\&\Diamond B), \Diamond(A\&B))$

(axiom-9) $Ka\ (\Diamond(A\&B), \Diamond A\&\Box B)$

If we restrict modal premises to the type-1, type-2, and type-3, then the above three axioms are sufficient. But if we want to make clear the concept of analyticity, or that of compatibility between terms, then we must add the following hypotheses;

(hyp-9) *The Modal Law of Contradiction*

$\phi\ (\Diamond(\overline{A}\&A))$

(hyp-10) *The Modal Law of Self-contradiction*

$\phi\ (\Diamond\ (A\&B)) \wedge \phi\ (\Diamond\ (A\&\overline{B})) \rightarrow \phi\ (\Diamond A)$

We can prove from (Df-12) (Df-13) (post-11) (hyp-9) that the modal law of the excluded middle holds good; *i. e.*

$\vdash\ t\ (\Box\ (A \text{or} \overline{A}))$

(hyp-9) means that $\overline{A}$ and A are always incompatible, while (hyp-10) means that if (A&B) and (A&$\overline{B}$) are incompatible, then A is self-contradictory.

We define four kinds of modal premises in the following way;

(Df-22)

(All B are necessarily A)	(All B are possibly A)
type-0: $A_a^{n_0}B \rightleftarrows \phi(\Diamond(\overline{A}\&B))$	$A_a^{p_0}B \rightleftarrows \phi(\Box(\overline{A}\&B))$
type-1: $A_a^{n_1}B \rightleftarrows \phi(\Diamond\overline{A}\&\Diamond B)$	$A_a^{p_1}B \rightleftarrows \phi(\Box\overline{A}\&\Diamond B)$
type-2: $A_a^{n_2}B \rightleftarrows \phi(\Diamond\overline{A}\&B)$	$A_a^{p_2}B \rightleftarrows \phi(\Box\overline{A}\&B)$
type-3: $A_a^{n_3}B \rightleftarrows \phi(\Diamond\overline{A}\&\Box B)$	$A_a^{p_3}B \rightleftarrows \phi(\Box\overline{A}\&\Box B)$
(No B are necessarily A)	(No B are possibly A)
$A_e^{n_k}B \rightleftarrows \overline{A}_a^{n_k}B$ $(0 \leqq k \leqq 3)$	$A_e^{p_k}B \rightleftarrows \overline{A}_a^{p_k}B$
(Some B are necessarily A)	(Some B are possibly A)
$A_i^{n_k}B \rightleftarrows \sim(A_e^{p_k}B)$	$A_i^{p_k}B \rightleftarrows \sim(A_e^{n_k}B)$
(Some B are necessarily not A)	(Some B are possibly not A)
$A_o^{n_k}B \rightleftarrows \sim(A_a^{p_k}B)$	$A_o^{p_k}B \rightleftarrows \sim(A_a^{n_k}B)$

We can prove the following theorems from (hyp-6) and the foregoing axioms;

$\vdash\ A_a^{n_1}B \rightleftarrows Ka(\Box A, \Diamond B)$ $\vdash\ A_a^{n_2}B \rightleftarrows Ka(\Box A, B)$ $\vdash\ A_a^{n_3}B \rightleftarrows Ka(\Box A, \Box B)$

$\vdash\ A_a^{p_1}B \rightleftarrows Ka(\Diamond A, \Diamond B)$ $\vdash\ A_a^{p_2}B \rightleftarrows Ka(\Diamond A, B)$ $\vdash\ A_a^{p_3}B \rightleftarrows Ka(\Diamond A, \Box B)$

(Df-23) *The Definition of Necessary Equivalence*

$A \overset{n_k}{\equiv} B \rightleftarrows (A_a^{n_k}B) \wedge (B_a^{n_k}A)$ $(0 \leqq k \leqq 3)$

$A \overset{n_k}{\equiv} B$: A is n_k-equivalent to B.

There is a consequential order among various types of necessary equivalence;

$$\begin{tikzcd} \vdash\ A\overset{n_1}{\equiv}B \arrow[r] & A\overset{n_2}{\equiv}B \arrow[r] \arrow[dr] & A\overset{n_3}{\equiv}B \\ & A\overset{n_0}{\equiv}B \arrow[r] \arrow[ur] & A\equiv B \end{tikzcd}$$

(Df-24) *The Definition of Possible Equivalence*

$A\overset{p_k}{\equiv}B \rightleftarrows (A^{p_k}_{a}B)\wedge(B^{p_k}_{a}A)$ $(0\leqq k\leqq 3)$

$A\overset{p_k}{\equiv}B$: A is p_k-equivalent to B.

There is also a consequential order among various types of possible equivalence;

$$\begin{tikzcd} \vdash\ A\overset{p_1}{\equiv}B \arrow[r] & A\overset{p_2}{\equiv}B \arrow[r] \arrow[d, leftrightarrow] & A\overset{p_3}{\equiv}B \\ & A\equiv B \arrow[r] & A\overset{p_0}{\equiv}B \end{tikzcd}$$

The following seven types of self-equivalence are provable in our system; $(0\leqq k\leqq 3)$

$\vdash\ A\overset{n_3}{\equiv}A \quad \vdash\ A\overset{n_0}{\equiv}A \quad \vdash\ A\equiv A \quad \vdash\ A\overset{p_k}{\equiv}A$

$(A\overset{n_1}{\equiv}A)$ and $(A\overset{n_2}{\equiv}A)$ don't hold good for every term in our system, but the following theorems generally hold;

$\vdash(A\overset{n_1}{\equiv}A)\rightarrow(\Box A\equiv A\equiv\Diamond A)$ *i. e.* if A is n_1-equivalent to itself, then the distinction of modality of A virtually disappears.

$\vdash(A\overset{n_2}{\equiv}A)\rightarrow(\Box A\equiv A)$ *i. e.* if A is n_2-equivalent to itself, then the distinction of necessity from actuality of A virtually disappears.

There is a kind of irregularity in the case of modal equivalence except n_0, n_3, and p_1 types. The following table shows that three laws which are valid in the case of simple equivalence don't always hold good in modal contexts;

(a) *The Law of Reflection*: $A\equiv A$
(b) *The Law of Symmetry*: $A\equiv B\rightarrow B\equiv A$
(c) *The Law of Transition*: $A\equiv B\wedge B\equiv C\rightarrow A\equiv C$

	$A\overset{n_0}{\equiv}B$	$A\overset{n_1}{\equiv}B$	$A\overset{n_2}{\equiv}B$	$A\overset{n_3}{\equiv}B$	$A\overset{p_0}{\equiv}B$	$A\overset{p_1}{\equiv}B$	$A\overset{p_2}{\equiv}B$	$A\overset{p_3}{\equiv}B$
(a)	0	—	—	0	0	0	0	0
(b)	0	0	0	0	0	0	0	0
(c)	0	0	0	0	—	0	—	—

(A-16) The definition of secondary substance

According Aristotile, the most distinctive mark of substance is that, while remaining numerically one and the same, it is capable of admitting contrary qualities: from among things other than substance, we should find ourselves unable to bring forward any which posseses this mark. (Cat. 4a10-15) While the primary substance plays the basic role in the context of existential presupposition, the secondary substance has a priority over attributes in the context of modal predication. We can use the concept of n_2-equivalence in order to define the secondary substance.

(Df-25) $S2(A)\rightleftarrows(\exists X)(ka(A, X))\wedge(\forall X)(\sim en(A, X))\wedge(A\overset{n_2}{\equiv}A)$
S2(A): A is a secondary substance.

If follows from the above definition that if A is a secondary substance and said of the subject B, then B cannot be otherwise than A;

i. e. $S2(A)\wedge Ka(A, B)\rightarrow Ka(\overline{\Diamond\bar{A}}, B)$

(A-17) The definition of contingency

The concept of contingency is defined in terms of that of possibility, while the concept of determination in terms of that of necessity.

(Df-26) $\langle c\rangle A=\Diamond A \,\&\, \Diamond\bar{A}$ $\qquad$ $\boxed{d}A=\Box A \text{ or } \Box\bar{A}$
$\langle c\rangle A$: contingent A $\qquad$ $\boxed{d}A$: determinant A

We can easily get from (Df-4) (Df-26) (post-11) the following theorems;

$\vdash\ \overline{\langle c\rangle A}=\boxed{d}\bar{A} \qquad \vdash\ \overline{\boxed{d}A}=\langle c\rangle\bar{A}$

$\vdash\ \langle c\rangle\bar{A}=\langle c\rangle A \qquad \vdash\ \boxed{d}\bar{A}=\boxed{d}A$

The determinant premises don't appear in Aristotle's texts, but it is convenient to introduce them on a par with contingency because of symmetrical relation.

(Df-27) *The Definition of Contingent and Determinant Premises*

(All B are contingently A) $\qquad$ (All B are determinantly A)
$A^{c_k}_{a}B\rightleftarrows A^{p_k}_{a}B\wedge A^{p_k}_{e}B$ $(0\leqq k\leqq 3)$ $\qquad$ $A^{d_k}_{a}B\rightleftarrows A^{n_k}_{a}B\vee A^{n_k}_{e}B$
(Some B are contingently A) $\qquad$ (Some B are determinantly A)
$A^{c_k}_{i}B\rightleftarrows A^{p_k}_{i}B\wedge A^{p_k}_{o}B$ $\qquad$ $A^{d_k}_{i}B\rightleftarrows A^{n_k}_{i}B\vee A^{n_k}_{o}B$

It is evident from the above definition that,
"No B are contingently A" is equivalent to "All B are contingently A" "Some B are contingently not A" is equivalent to "Some B are contingently A." *i. e.*

$A^{c_k}_{e}B\rightleftarrows A^{c_k}_{a}B \qquad A^{c_k}_{o}B\rightleftarrows A^{c_k}_{i}B$

Aristotle points out this kind of equivalence. (An. Pr. A 13-32a)

Similarly the following kinds of equivalence hold good;

$\vdash\ A^{d_k}_{e}B\rightleftarrows A^{d_k}_{a}B \qquad \vdash\ A^{d_k}_{o}B\rightleftarrows A^{d_k}_{i}B$

$\vdash\ \sim(A^{d_k}_{e}B)\rightleftarrows A^{c_k}_{i}B \qquad \vdash\ \sim(A^{c_k}_{e}B)\rightleftarrows A^{d_k}_{i}B \quad (0\leqq k\leqq 3)$

The contingent premises can be rewritten as follows;

$\vdash\ A^{c_1}_{a}B\rightleftarrows Ka(\langle c\rangle A, \Diamond B) \quad \vdash\ A^{c_2}_{a}B\rightleftarrows Ka(\langle c\rangle A, B) \quad \vdash\ A^{c_3}_{a}B\rightleftarrows Ka(\langle c\rangle A, \Box B)$

We further add the type-4 of contingent premises as follows;

(Df-28)
$A^{c_4}_{a}B\rightleftarrows A^{c_4}_{e}B\rightleftarrows Ka(\langle c\rangle A, \langle c\rangle B)$
$A^{c_4}_{i}B\rightleftarrows A^{c_4}_{o}B\rightleftarrows\sim Ka(\boxed{d}A, \langle c\rangle B)$

The characteristic of c_4-contingent premises is that O-conversion holds good while E-conversion doesn't hold good. (*cf.* An. Pr. 25b17)

We can use the concept of contingency in order to define accidental attributes.

(*cf.* Topica 102b)

(Df-29) $Ac_k(A, B) \rightleftarrows A\,{}^{ck}_{1}\,B \qquad (0 \leqq k \leqq 4)$

$Ac_k(A, B)$: A is an accidental attribute of B.

We can also use the concept of modal equivalence in order to define property (to idion) as follows;

(Df-30) $P(A, B) \rightleftarrows (Ps(B) \vee S2(B)) \wedge (A \equiv B) \wedge (A \neq B)$

$P(A, B)$: A is B's property.

$Pn_k(A, B) \rightleftarrows (Ps(B) \vee S2(B)) \wedge (A \overset{nk}{\equiv} B) \wedge (A \neq B)$

$Pn_k(A, B)$: A is B's necessary property.

$Pp_k(A, B) \rightleftarrows (Ps(B) \vee S2(B)) \wedge (A \overset{pk}{\equiv} B) \wedge (A \neq B)$

$Pp_k(A, B)$: A is B's possible property.

(A-18) The modality of existence

We introduce a constant term *E to our system in order that we may designate the modality of existence;

(hyp-11) $Ka({*E}, A) \rightleftarrows Exist(A)$

Then we apply three term operators $\Diamond$, $\Diamond\hspace{-0.55em}c\;$ and $\Box$ to *E in order to define respective types of modal existence;

(Df-31)

$P\text{-}Exist(A) \rightleftarrows Ka(\Diamond {*E}, A)$

P-Exist(A): A is a possible entity.

$C\text{-}Exist(A) \rightleftarrows Ka(\Diamond\hspace{-0.55em}c\; {*E}, A)$

C-Exist(A): A is a contingent entity.

$N\text{-}Exist(A) \rightleftarrows Ka(\Box {*E}, A)$

N-Exist(A): A is a necessary entity.

The modality of existence is crucial to the problems of metaphysica generalis: the so-called argument from contingency has been a controvercial issue to theologians. These problems, however, will not be discussed in this paper, of which the theme is restricted to general ontology.

(B) Applications of Our System to Some Exegetical Problems

(B-1) Whether Aristotle's two-term theory of subject and predicate commited an original sin in the history of logic?

We proceed to the first question in the following way:

Obj. 1.—P. T. Geach insists that Aristotle's thesis on the interchangeablity of subject and predicate is incurably fallacious. He says, "it is logically impossible for a term to shift about between subject and predicate position without undergoing a change of sense as well as a change of role." According to him only a name can be a logical subject; and a name cannot retain the role of a name if it becomes a

logical predicate; for a predicate purports to give us what holds or doesnot hold good of an individual, but a name just serves to name or refer to an individual. He concludes that Aristotle dropped the requirement that a subject of predication must be a syntactically simple name, and went over to the two-term theory, which is a disaster, "comparable to the Fall of Adam."[13]

Obj. 2.—Bertrand Russell blamed Aristotle in the same vein for his having failed to see the type-difference of subject from predicate: he said, "Another error into which Aristotle falls through this mistake is to think that a predicate of a predicate can be a predicate of the original subject. If I say "Socrates is Greek, all Greeks are human," Aristotle thinks that "human" is a predicate of "Greek," while "Greek" is a predicate of "Socrates," and obviously "human" is a predicate of "Socrates." But in fact "human" is not a predicate of "Greek." The distinction between names and predicates, or in metaphysical language, between particulars and universals, is thus blurred, with disastrous consequences to philosophy. One of the resulting confusions was to suppose that a class with only one member is identical with that one member. This made it impossible to have a correct theory of the number one, and led to endless bad metaphysics about unity."[14]

On the contrary,

the axiomatic set theory (Zermelo-Fraenkel) uses the idea of two-term theory which Geach repudiates as fallacious. In this theory sets are treated as nameable entities on a par with individuals; a set of a set can be a set of the original element. Moreover an individual can be identified as the class whose sole member is that individual.[15]

I answer that,

logicians must be tolerant to different views of ontology. Both Geach and Russell argues against Aristotle on the basis of Frege's paradigm; Frege expressed the subject-predicate dichotomy in the argument-function notation in mathematical logic.[16] Therefore the proposition assumes such functional forms as F(x), and predicates are turned into propositional functions.[17] Once this paradigm is accepted, the famous principle of classical logic that nota notae est etiam nota rei ipsius doesnot hold. For we are prohibited to use such formulae as F(F) or as x(F). So far as we retain Frege's paradigm, we are forced to say that subject and predicate are not convertible, and a predicate of a predicate cannot be a predicate of the original subject. But Frege's is not the only paradigm of logic. Argument-function notation need not correspond to the subject-predicate dichotomy. We can treat both subject and predicate as two arguments of the function which corresponds to the cpula "be". This is a kind of two-term theory, which Geach dogmatically rejects, but such logicians as Lesniewski, Zermelo and Fraenkel, adopted in their respective types

[13] P. T. Geach, "History of the Corruption of Logic" (*Logic Matters*, pp. 44–61).

[14] B. Russell, "Aristotle's Logic" (*History of Western Philosophy*, pp. 206–212.).

[15] W. V. O. Quine, *Set Theory and its Logic*, pp. 31.

[16] G. Frege, "Function and Concept" (*Philosophical Writings of G. Frege*, pp. 21–41).

[17] B. Russell, "The Logical and Arithmetical Doctrines of Frege" (*Principles of Mathematics*, pp. 501–528).

of set theory or ontology.[18] Aristotle's ontology can be axiomatized in the same way as a set theory can. But there is a great difference between Aristotle's ontology and the set theory. Sets are purely extensional entities and have only to have enumerative unity, while such entities as genus, species, and attributes in Airstotelian ontology, are intensional ones. There are many problems which are peculiar to such a type of ontology but need not be dealt with in the set theory; the criterion of equality or of equvalence is notoriously a difficult one. Our system contains Ka(A, B) and En(A, B) as basic formulae, which correspond respectively to predicative and existential uses of the verb "be." Ontology which literally means a discourse of being is possible only when such tripartite divisions are permitted.

(B-2) Whether the theory of syllogism commits a fallacy of petitio principii or not?

We proceed to the second question in the following way;

Obj. 1.—It seems that the syllogism is invalid in our everyday discourse. "All men are mortal. Socrates is a man. Therefore, Socrates is mortal." This is a textbookish example of syllogism; but what does "therefore" mean in the above inference? Suppose that there were no other man than Socrates and Plato; then the syllogism in question would run as follows: "Socrates and Plato are mortal. Socrates is Socrates or Plato. Therefore, Socrates is mortal." What a curious inference that would be! It proves to be nothing more than a tautology, in which "therefore" plays no role at all.[19]

Obj. 2.—It seems that the syllogism is useless in empirical sciences; for how can we know that all men are mortal? The universal premise cannot be proved to be true on purely empirical grounds. Therefore, if we are to claim that the syllogism is something more than a tautology, we must say that the universal premises are, strictly speaking, nothing other than hypotheses which are susceptible of refutation through unexpected empirical data. The syllogistic necessity is only a hypothetical one, though Aristotle seemed to think that the premises of the apodeictic syllogism must be more certain (*i. e.* intelligible per se) than a conclusion.[20]

On the contrary,

Hischberger points out that the syllogism is a part of metaphysics, and should not be explained away on the basis of positivism or of pragmatism.[21]

[18] A. Ishimoto, "A Lesniewskian Version of Montague Grammar", *Proceedings of the ninth International Conference on Computational Linguistics*, Prague, 1982, pp. 139–144.

[19] *cf.* Sextus Empiricus, *Hyp. Pyrrh.* 2–164.

[20] K. Popper, *Conjectures and Refutations*, pp. 215–250. Popper claims that scientific principles should be considered as refutable hypotheses which are more "improbable" than individual data of observation. "Science aims at improbability" is the result of his arguments. But this view of scientific theories, though it contains many valuable insights, is contrary to the use of "improbable" in the ordinary discourse, and even to the actual situation of scientific research. It is true that the theories which had been believed to be most certain were replaced by others in the history of science, but this is to be considered as a kind of paradigm change rather than as any advance of what he calls verisimilitude, *i. e.* high improbability plus more truth content. Scientific theories are neither tautologies nor so many pseudo-explanations of "certain" data by uncertain or "improbable" hypotheses.

[21] J. Hirschberger, *Geschichte der Philosophie*, I Alteltum, ch. 2-A.

I answer that,

we must be reminded of the original form of Aristotelian syllogism; according to Prior Analytics it runs as follows:

"If A is predicated of every B, and B of every C, then necessarily A is predicated of every C." (ei gar to A kata pantos tou B kai to B kata pantos tou Γ, anagkê to A kata pantos tou Γ katêgoreîsthai.) (25b37-39) This is very different from the well-known formulation, "All B are A. All C are B. Therefore, all C are A." First, Aristotle's own formula asserts something conditionally about all entities presupposed in his ontology, while the so-called syllogistic inference asserts something unconditionally about all statements of a certain kind. If A and B designates respectively a genus and a species, then we can say categorically, "All B are A." But in other cases only the original hypothetical formula may hold good. Second, the original formula that A is predicated of every B must be distinguished from the categorical statement that all B are A, when B=A&D. According to the notation of our system Ka (A, A&D) is true without any existential condition. But when we ask whether A and B has a real unity, *i. e.* whether (A&B) exists, we must have some criterion of existence. Aristotle was evidently aware of that situation, when he said (An. Post. 76b-35), "Definitions are not hypotheses, because they make no assertion of existence or non-existence." On the other hand, the categorical statement that all B are A has an existential import; it is expressed as Aa′B, *i. e.* Ka(A, B)∧ Exist(B), in our system.

I agree to Hirschberger's opinion that Aristotelian syllogism should be considered as a part of metaphysics, and have tried a reconstruction of the syllogistic on the basis of general ontology.

There is a fundamental presupposition in Aristotelian ontology that the genus has more than an enumerative unity; it has a real unity, and provides the syllogism with a real basis, which is often absent from many versions of the so-called classical logic. When we say, "(Being) mortal is said of the subject man", we need not presuppose the existence of Socrates; "man" designates a species, a secondary substance, which has, as we have seen in (A-16), a kind of priority over individual instances. Therefore the syllogism doesnot commit a fallacy of petitio principii in any sense.

(B-3) Whether Aristotle's theory of modal syllogism can be turned to a consistent system or not?

We proceed to the third question in the following way;

Obj.—It seems that Aristotle's system cannot be made consistent; for he accepts the conversion rules such as $A^{p}_{e}B \rightleftarrows B^{p}_{e}A$ and $A^{n}_{e}B \rightleftarrows B^{n}_{e}A$, while he doesn't reject such syllogisms as B^{p}_{a}rbar$^{p}_{a}$ and B^{n}_{a}rbar$^{n}_{a}$. For example, J. Hintikka cites a passage from Prior Analytics (33b34-6):

(a) A possibly applies to all B. (the major premise)
(b) B applies to all C. (the minor premise)
(c) A possibly applies to all C. (the conclusion)

Then he says, "No matter how you try to interprete the premises, there is no hope of turning the syllogism into a valid one unless you somehow lend modal (apodeictic) force to (c) also." He concludes, "Those discussions of Aristotle's modal syllogistic that concentrate on systematizing the syllogisms (syllogistic modes) accepted by Aristotle are completely misplaced."[22]

I answer that,
the only way in which we can rescue Aristotle from inconsistency is to discriminate carefully the ambiguities of modal premises.

We can deduce from (a) and (b) the conclusion (c), if we rewrite them as follows;

(a) $A^{p_2}_{a}B$ (*i. e.* Ka(◇A, B)) (b) $B_{a}C$ (*i. e.* Ka(A, B))
(c) $A^{p_2}_{a}C$ (*i. e.* Ka(◇A, C))

It is very natural for Aristotle to call this type of syllogism "complete"; for it depends on (axiom-1) in our system. Though the conversion rule, "$A^{p_2}_{e}B \rightleftarrows B^{p_2}_{e}A$", must be rejected, we can prove in our system that $A^{p_3}_{e}B \rightleftarrows B^{p_3}_{e}A$ and $A^{p_0}_{e}B \rightleftarrows B^{p_0}_{e}A$. Aristotle relies on different principles in different parts of his modal syllogistic; for example, he discriminates the different structures of possible premises which correspond to $A^{p_1}_{a}B$, and $A^{p_2}_{a}B$ in our system. (An. Pr. 32b25-32)

I admit that Hintikka's view is right when he points out that axiomatizing a set whose membership is based on sheer accident is a pointless exercise. This is in fact a leading principle of our system; we don't accept Barbara and the conversion rule axiomatically in the case of modal syllogistic. We set up the axioms in terms of modal term-operators and Ka(A, B), and then define the various types of modal premises. Our system aims at showing that Aristotle's concept of modality does not depend on sheer accident but has a focal meaning, which can be expressed as an axiomatized system.

(B-4) Whether the law of substitutivity of identity salva veritate holds good in modal logic or not?

We proceed to the fourth question in the following way;
Obj.—It seems that the law of substitutivity of identity salva veritate does not hold good in modal logic; for if we admit it, then the following examples would be admitted as valid, though they are intuitively fallacious.[23]

(ex-1)
(a) Necessarily 9 is (identical with) 9. (the major premise)
(b) The number of planets is (identical with) 9. (the minor premise)

(c) Necessarily the number of planets is (identical with) 9. (the conclusion)

(ex-2)
(a) The first president of U. S. A. necessarily exists. (the major premise)
(b) Washington is (identical with) the first predisent of U. S. A.. (the minor premise)

(c) Washington necessarily exists. (the conclusion)

[22] J. Hintikka, *Time and Necessity*, pp. 136.
[23] W. V. Quine, *From a Logical Point of View*, pp. 143.

I answer that,
the law of substitutivity holds good only in the case of equality which is defined by (Df-1), while it does not always hold good in the various cases of equivalence which are defined by (Df-23) and (Df-24) in our system.

According to our notation, the first example is turned to the following reasoning;

(a) $9 \overset{n_1}{\equiv} 9$ (or $9 \overset{n_0}{\equiv} 9$) (the major premise)
(b) the number of planets$\equiv$9 (the minor premise)

(c) the number of planets$\overset{n_1}{\equiv}$9
(or the number of planets$\overset{n_0}{\equiv}$9) (the conclusion)

The above scheme of reasoning is not valid in our system; the problem of intensional identity is solved by analysing the ambiguities of equivalence. We must notice that even the substitutivity of necessary equivalence does not always hold good in our system; this corresponds to Aristotelian distinction between necessary properties and essential definitions.

The second example requires a little more subtle consideration. "Necessity" which is mentioned in (ex-2-a) is a typical example of modality de dicto; we can rewrite it to the following hypothetical statement: "If there is a system of presidency in U. S. A., then necessarily there is the first president of U. S. A.."
In this context "necessarily" designates only a conditional one, and does not represent anybody's property or essence. (*cf.* E. N. 1096a20—Aristotle's dictum that substance precedes relations) On the contrary, "necessity" which is mentioned in the conclusion of (ex-2) may be rewritten as follows; N-Exist (Washington). But this turns to be false because Washington is a sensible substanse with contingent modality of existence.

(Addenda) Barbara and the Rules of Conversion with Modality

((PL) designates the rule of propositional inference)

The Proof of n_0-Barbara

(1) $\phi(\Diamond(\bar{A}\&B)) \rightarrow \phi(\Diamond(\bar{A}\&C\&B))$ ...{(post-4) (post-6) (Df-13) (PL)
(2) $\phi(\Diamond(\bar{B}\&C)) \rightarrow \phi(\Diamond(\bar{A}\&C\&\bar{B}))$ ...{(axiom-1) (axiom-4) (axiom-8)
(3) $\phi(\Diamond(\bar{A}\&B)) \wedge \phi(\Diamond(\bar{B}\&C)) \rightarrow \phi(\Diamond(\bar{A}\&C\&B)) \wedge \phi(\Diamond(\bar{A}\&C\&\bar{B}))$ ...(1) (2) (PL)
(4) $\phi(\Diamond(\bar{A}\&C\&B)) \wedge \phi(\Diamond(\bar{A}\&C\&\bar{B})) \rightarrow \phi(\Diamond(\bar{A}\&C))$ ...(hyp-10)
(5) $\phi(\Diamond(\bar{A}\&B)) \wedge \phi(\Diamond(\bar{B}\&C)) \rightarrow \phi(\Diamond(\bar{A}\&C))$ ...(3) (4) (PL)
(6) $A^{n_0}_{a}B \wedge B^{n_0}_{a}C \rightarrow A^{n_0}_{a}C$ ...(5) (Df-22)

The Proof of n_0-Celarent

(1) $\bar{A}^{n_0}_{a}B \wedge B^{n_0}_{a}C \rightarrow \bar{A}^{n_0}_{a}C$ ...(n_0-Barbara)
(2) $A^{n_0}_{e}B \wedge B^{n_0}_{a}C \rightarrow A^{n_0}_{e}C$ ...(1) (Df-22)

The Proof of n_0-Darii

(1) $\phi(\Diamond(\bar{A}\&B)) \rightarrow \phi(\Diamond\bar{A}\&\Box B)$ ...(axiom-9) (Df-13)

(2) $\phi(\Diamond\bar{A}\&\Box B)\rightleftarrows Ka(\Box A, \Box B)\rightleftarrows Ka(\overline{\Box B}, \overline{\Box A})$ ...(post-2) (post-11) (hyp-6) (axiom-6) (PL)

(3) $Ka(\overline{\Box B}, \overline{\Box A})\wedge Ka(\overline{\Box A}, \Box C)\rightarrow Ka(\overline{\Box B}, \Box C)$ (axiom-1) (PL)

(4) $A^{n_0}_{a}B\rightleftarrows\phi(\Diamond(\bar{A}\&B))$ ...(Df-22)

(5) $A^{p_0}_{e}C\rightleftarrows A^{p_3}_{e}C\rightleftarrows Ka(\Diamond\bar{A}, \Box C)\rightleftarrows Ka(\overline{\Box A}, \Box C)$ ...(post-2) (post-11)

(6) $B^{p_0}_{e}C\rightleftarrows B^{p_3}_{e}C\rightleftarrows Ka(\Diamond\bar{B}, \Box C)\rightleftarrows Ka(\overline{\Box B}, \Box C)$ (hyp-6) (Df-22)

(7) $A^{n_0}_{a}B\rightarrow(A^{p_0}_{e}C\rightarrow B^{p_0}_{e}C)$ ...(3) (4) (5) (6) (PL)

(8) $A^{n_0}_{a}B\rightarrow(\sim B^{p_0}_{e}C\rightarrow\sim A^{p_0}_{e}C)$ ...(7) (PL)

(9) $A^{n_0}_{a}B\rightarrow(B^{n_0}_{i}C\rightarrow A^{n_0}_{i}C)$ ...(8) (Df-22)

(10) $A^{n_0}_{a}B\wedge B^{n_0}_{i}C\rightarrow A^{n_0}_{i}C$ ...(9) (PL)

The Proof of n_0-Ferio

(1) $\bar{A}^{n_0}_{a}B\wedge B^{n_0}_{i}C\rightarrow\bar{A}^{n_0}_{i}C$ ...(n_0-Darii)

(2) $A^{n_0}_{e}B\wedge B^{n_0}_{i}C\rightarrow A^{n_0}_{o}C$ ...(1) (Df-22)

The Proof of n_0-E-conversion

$A^{n_0}_{e}B\rightleftarrows\phi(\Diamond(A\&B))\rightleftarrows\phi(\Diamond(B\&A))\rightleftarrows B^{n_0}_{e}A$

The Proof of n_0-I-conversion

$A^{n_0}_{i}B\rightleftarrows\sim(\bar{A}^{p_0}_{a}B)\rightleftarrows\sim\phi(\Box(A\&B))\rightleftarrows\sim\phi(\Box(B\&A))\rightleftarrows\sim(\bar{B}^{p_0}_{a}A)\rightleftarrows B^{n_0}_{i}A$

The Proof of accidental n_0-conversion

(Df) $A^{n_0:}_{a}B\rightleftarrows A^{n_0}_{a}B\wedge B^{n_0}_{i}B$

(1) $A^{n_0}_{a}B\wedge B^{n_0}_{i}B\rightarrow A^{n_0}_{i}B$ ...(n_0-Darii)

(2) $A^{n_0:}_{a}B\rightarrow A^{n_0}_{i}B$ ...(1) (Df)

(3) $A^{n_0}_{i}B\rightleftarrows B^{n_0}_{i}A$ ...(n_0-I-conversion)

(4) $A^{n_0:}_{a}B\rightarrow B^{n_0}_{i}A$ ...(2) (3) (PL)

The Proof of n_1-Barbara

(1) $Ka(\Diamond B, \Box B)$ ...(axiom-1) (axiom-6) (axiom-7) (PL)

(2) $Ka(\Diamond B, \Box B)\wedge Ka(\Box B, \Diamond C)\rightarrow Ka(\Diamond B, \Diamond C)$ ...(axiom-1)

(3) $Ka(\Box B, \Diamond C)\rightarrow Ka(\Diamond B, \Diamond C)$ ...(1) (2) (PL)

(4) $Ka(\Box A, \Diamond B)\wedge Ka(\Diamond B, \Diamond C)\rightarrow Ka(\Box A, \Diamond C)$ ...(axiom-1)

(5) $Ka(\Box A, \Diamond B)\wedge Ka(\Box B, \Diamond C)\rightarrow Ka(\Box A, \Diamond C)$ ...(3) (4) (PL)

(6) $A^{n_1}_{a}B\wedge B^{n_1}_{a}C\rightarrow A^{n_1}_{a}C$ ...(5) (Df-22)

The Proof of n_1-E-conversion

$A^{n_1}_{e}B\rightleftarrows\phi(\Diamond A\&\Diamond B)\rightleftarrows\phi(\Diamond B\&\Diamond A)\rightleftarrows B^{n_1}_{e}A$

The Proof of n_2-Barbara

(1) $Ka(\Box A, B)\wedge Ka(B, C)\rightarrow Ka(\Box A, C)$ ...(axiom-1)

(2) $Ka(B, \Box B)$ ...(axiom-6) (axiom-7) (PL)

(3) $Ka(B, \Box B)\wedge Ka(\Box B, C)\rightarrow Ka(B, C)$ ...(axiom-1)

(4) $Ka(\Box B, C)\rightarrow Ka(B, C)$ ...(2) (3) (PL)

(5) $Ka(\Box A, B)\wedge Ka(\Box B, C)\rightarrow Ka(\Box A, C)$ ...(1) (4) (PL)

(6) $A^{n_2}_{a}B\wedge B^{n_2}_{a}C\rightarrow A^{n_2}_{a}C$ ...(5) (Df-22)

The Proof of n_3-Barbara

(1) $Ka(\Box A, \Box B)\wedge Ka(\Box B, \Box C)\rightarrow Ka(\Box A, \Box C)$ ...(axiom-1)

(2) $A^{n_3}_{a}B\wedge B^{n_3}_{a}C\rightarrow A^{n_3}_{a}C$ ...(1) (Df-22)

The Proof of n_3-I-conversion

$A^{n_3}_{i}B\rightleftarrows\sim(\bar{A}^{p_3}_{a}B)\rightleftarrows\sim\phi(\Box A\&\Box B)\rightleftarrows\sim\phi(\Box B\&\Box A)\rightleftarrows\sim(\bar{B}^{p_3}_{a}A)\rightleftarrows B^{n_3}_{i}A$

The Proof of p_0-E-conversion

$A^{p_0}_{e}B\rightleftarrows\phi(\Box(A\&B))\rightleftarrows\phi(\Box(B\&A))\rightleftarrows B^{p_0}_{e}A$

The Proof of p_0-I-conversion

$A^{p_0}_{i}B\rightleftarrows\sim\phi(\Diamond(A\&B))\rightleftarrows\sim\phi(\Diamond(B\&A))\rightleftarrows B^{p_0}_{i}A$

The Proof of p_1-Barbara

(1) $Ka(\Diamond A, \Diamond B)\wedge Ka(\Diamond B, \Diamond C)\rightarrow Ka(\Diamond A, \Diamond C)$ ...(axiom-1)

(2) $A^{p_1}_{a}B\wedge B^{p_1}_{a}C\rightarrow A^{p_1}_{a}C$ ...(1) (Df-22)

The Proof of p_1-I-conversion

$A^{p_1}_{i}B\rightleftarrows\sim(A^{n_1}_{e}B)\rightleftarrows\sim\phi(\Diamond A\&\Diamond B)\rightleftarrows\sim\phi(\Diamond B\&\Diamond A)\rightleftarrows\sim(B^{n_1}_{e}A)\rightleftarrows B^{p_1}_{i}A$

The Proof of p_3-E-conversion

$A^{p_3}_{e}B\rightleftarrows\phi(\Box A\&\Box B)\rightleftarrows\phi(\Box B\&\Box A)\rightleftarrows B^{p_3}_{e}A$

The Proof of c_1-Barbara

(1) $Ka(\Diamond B, \Diamond_c B)$ ...(Df-22) (axiom-4)

(2) $Ka(\Diamond B, \Diamond_c B)\wedge Ka(\Diamond_c B, \Diamond C)\rightarrow Ka(\Diamond B, \Diamond C)$ ...(axiom-1)

(3) $Ka(\Diamond_c B, \Diamond C)\rightarrow Ka(\Diamond B, \Diamond C)$ ...(1) (2) (PL)

(4) $Ka(\Diamond_c A, \Diamond B)\wedge Ka(\Diamond B, \Diamond C)\rightarrow Ka(\Diamond_c A, \Diamond C)$ ...(axiom-1)

(5) $Ka(\Diamond_c A, \Diamond B)\wedge Ka(\Diamond_c B, \Diamond C)\rightarrow Ka(\Diamond_c A, \Diamond C)$ ...(3) (4) (PL)

(6) $A^{c_1}_{a}B\wedge B^{c_1}_{a}C\rightarrow A^{c_1}_{a}C$ ...(5) (Df-27) (axiom-4) (axiom-5) (PL)

The Proof of c_4-Barbara

(1) $Ka(\Diamond_c A, \Diamond_c B)\wedge Ka(\Diamond_c B, \Diamond_c C)\rightarrow Ka(\Diamond_c A, \Diamond_c C)$ ...(axiom-1)

(2) $A^{c_4}_{a}B\wedge B^{c_4}_{a}C\rightarrow A^{c_4}_{a}C$ ...(1) (Df-28)

The Proof of c_4-o-conversion (*i. e.* c_4-I-conversion)

$A^{c_4}_{o}B\rightleftarrows A^{c_4}_{i}B\rightleftarrows\sim Ka(\boxed{d}A, \Diamond_c B)\rightleftarrows\sim Ka(\boxed{d}\bar{A}, \Diamond_c\bar{B})\rightleftarrows\sim Ka(\overline{\Diamond_c B}, \overline{\boxed{d}A})$
$\rightleftarrows\sim Ka(\boxed{d}B, \Diamond_c A)\rightleftarrows B^{c_4}_{i}A\rightleftarrows B^{c_4}_{o}A$

The Examples of irregular Conversion

$A^{p_1}_{e}B\rightleftarrows\phi(\Box A\&\Diamond B)\rightleftarrows\phi(\Diamond B\&\Box A)\rightleftarrows B^{n_3}_{e}A$

$A^{n_1}_{i}B\rightleftarrows\sim(A^{p_1}_{e}B)\rightleftarrows\sim(B^{n_3}_{e}A)\rightleftarrows B^{p_3}_{i}A$

	n_0	n_1	n_2	n_3	p_0	p_1	p_2	p_3	c_0	c_1	c_2	c_3	c_4
Barbara	0	0	0	0	—	0	—	—	—	0	—	—	0
E-conversion	0	0	—	△	0	△	—	0	—	—	—	—	—
I-conversion	0	△	—	0	0	0	—	△	—	—	—	—	0

("△" designates an irregular conversion)